Acceleration of dust particles by low-frequency Alfvén waves

V. Prudskikh [a], Yu. A. Shchekinov [b]

[a] *Institute of Physics, South Federal University, Stachki St. 194*

[b] *Department of Physics, South Federal University, Sorge St. 5, Rostov-on-Don, 344090 Russia*

Abstract

We investigate the efficiency of acceleration of charged dust particles by low-frequency Alfvén waves in nonlinear approximation. We show that the longitudinal acceleration of dust particles is proportional to the square of the soliton amplitude $O(|b_m|^2)$, while the transversal acceleration is of $O(|b_m|)$. In the conditions of the interstellar medium the resulting velocity of dust particles can reach 0.3 to 1 km s^{-1}.

Key words:

1 Introduction

The presence of charged dust grains in space plasma may play crucial influence on the overall dynamics, as for instance, the breaking down of frozen-in-field conditions [1] and dissipation of magnetic flux [2] in the interstellar molecular clouds. The dynamical importance of dust grains, and circulation of their mass (dust destruction and growth) in the interstellar medium is essentially determined by their kinetic temperature, and therefore understanding of how they are accelerated is of significant interest [3,4]. In general, magnetohydrodynamic (MHD) waves are known as an efficient heating source of plasma components. For instance, interaction of dispersive Alfvén waves with plasma results in Joule heating of the electrons, generation of zonal flows and formation of localized Alfvén structures and Alfvén vortices [5]. In a dusty plasma MHD waves, in particular, compressible fast MHD modes, are found to be efficient in heating of translational degrees of freedom of dust particles through

Email addresses: `E-mail:slavadhb@mail.ru` (V. Prudskikh), `E-mail:yus@phys.rsu.ru` (Yu. A. Shchekinov).

gyro-resonant acceleration [3]. More recently ponderomotive forces of the shear Alfvén waves are shown to be a powerfull mechanism of dust acceleration [6], such that in a short time a microsized dust particle of mass $m_d \sim 10^{-12}$ g can reach a few km s^{-1} velocity – comparable to the phase velocity of the Alfvén wave. In [6] a steady nonuniform field envelope with a fixed field gradient (caused, for instance, by a nonuniform plasma density distribution along the wave vector) is described. Contrary, in our analyses below the ponderomotive force is unsteady in the rest reference frame where the pulse envelope moves with the group velocity. In this approach we are able to derive the perturbation amplitudes in the reference frame connected with the pulse, for densities and longitudinal velocities of plasma components driven by the mutual action of Coulomb and ponderomotive forces. Therefore, the nonuniformity of the field amplitude and the corresponding plasma acceleration in the pulse follows in our approach from a self-consistent solution of MHD equations.

In [6] they considered a relatively high-frequency limit, corresponding to frequencies much higher than the dust cyclotron frequency ω_d. It might be expected though that in the low-frequency end $\omega \ll \omega_d$ the presence of dust can have an important influence on dynamics of Alfvén waves. In particular, one can think that if dust particles are accelerated, the energy gained by them from the Alfvén wave can be sufficient to decrease its amplitude, which in turn can reduce the acceleration efficiency. In this paper we explore the efficiency of acceleration of dust grains by the nonlinear low-frequency Alfvén waves, and show that the nonlinearity strongly restricts the ability of Alfvén waves to transfer their energy to charged dust grains. Such solitons can be actually treated as elementary objects of MHD turbulence in the low-frequency range. We follow therefore recent discussion of interaction of such low-frequency Alfvén solitons with the plasma [7,8].

In the conditions of the interstellar plasma this low-frequency range, $\omega \ll \omega_d \sim 3 \times 10^{-11}$ Hz, can be of great interest from the point of view of dust acceleration because a considerable energy fraction of the interstellar turbulence can be locked in low-frequency motions [9]. Of course, nonlinear interactions move low-frequency Alfvén perturbations toward higher frequences. However, it seems natural to assume that in the interstellar environment a steady state spectrum of perturbations establishes on a relatively short time scale (definitely shorter than 100 Myr – the crossing time for a spiral wave), so that perturbations are always present in the low-frequency end.

We proceed as follows. In Section 2 we find the dispersion relation and the group velocities for longitudinal Alfvén and fast MHD waves, in Section 3 we derive the nonlinear wave equation; Section 4 contains estimates of the velocities of dust grains accelerated by the Alfvén and fast MHD soliton waves; summary of the results is given in Section 5.

2 Group velocity

In a fully ionized dusty plasma the dispersion relation for longitudinal Alfvén and fast MHD modes can be written as [12]

$$N^2 = \epsilon_1 \pm \epsilon_2, \tag{1}$$

where $N = kc/\omega$,

$$\epsilon_1 = 1 - \sum_\alpha \frac{\omega_{p,\alpha}^2}{\omega^2 - \omega_\alpha^2}, \;\; \epsilon_2 = -\sum_\alpha \frac{\omega_{p\alpha}^2 \omega_\alpha}{\omega(\omega^2 - \omega_\alpha^2)}, \tag{2}$$

$\alpha = e,\ i,\ d$, $\omega_{p\alpha}$ and ω_α are the plasma and cyclotron frequencies of the αth component. In the low-frequency limit $\omega \ll \omega_d$ equation (1) can be written in the form

$$\frac{\omega^2}{k^2} = v_A^2 \left[1 + \frac{\rho_d}{\rho_i} \pm \frac{\omega}{\omega_i}\left(1 - \frac{m_d \rho_d}{Z_d m_i \rho_i}\right)\right]^{-1}, \tag{3}$$

where signs $\pm$ belong to the Alfvén and fast MHD waves, respectively, $v_{Ai} = B_0/\sqrt{4\pi\rho_i}$ is the Alfvén speed.

Assuming $m_d\rho_d/Z_d m_i \rho_i \gg 1$ and $\omega \ll \rho_d\omega_d/\rho$, $\rho = \rho_i + \rho_d$, one can get from here

$$\omega^2 = k^2 v_A^2 \left(1 \pm \frac{\rho_d}{\rho} k r_{Ad}\right), \tag{4}$$

where $v_A^2 = B_0^2/4\pi\rho$, $r_{Ad} = v_A/\omega_d$. The group velocity and its derivative for Alfvén waves are

$$u = \frac{d\omega}{dk} = v_A \left(1 + \frac{\rho_d}{\rho} r_{Ad} k\right), \;\; \frac{du}{dk} = \frac{\rho_d}{\rho} v_A r_{Ad} > 0, \tag{5}$$

and for fast MHD mode

$$u = \frac{d\omega}{dk} = v_A \left(1 - \frac{\rho_d}{\rho} r_{Ad} k\right), \;\; \frac{du}{dk} = -\frac{\rho_d}{\rho} v_A r_{Ad} < 0. \tag{6}$$

3 Nonlinear parabolic equation for low-frequency Alfvén and fast MHD waves

Following the procedure described in [10,11] one can derive the nonlinear Schrödinger equation for the envelope of the Alfvén wave package in the form

$$\pm i\frac{\partial E_1}{\partial \tau} + \frac{u'}{2}\frac{\partial^2 E_1}{\partial \xi^2} - \frac{u}{2kc^2}\sum_k \frac{\omega_{pk}^2\omega}{\omega+\omega_k}\left(N_k - \frac{kv_{zk}}{\omega}\frac{\omega_k}{\omega+\omega_k}\right)E_1 = 0, \tag{7}$$

where

$$u' = \pm\frac{\rho_d}{\rho_i+\rho_d} r_{AD} v_A, \tag{8}$$

is the dispersion of the group velocity; sign "plus" belongs to the left-polarized, "minus" to the righ-polarized waves, $N_k = \tilde{n}_k/n_{k0}$ is the normalized density perturbation of kth component. In the left-polarized wave ω_k is positive for the electrons and dust particles, and negative for the ions, in the right-polarized Alfvén mode ω_k is positive for the ions and negative for the electrons and dust particles.

Assuming $v_{Ti} \ll v_A \ll v_{Te}$ one can neglect the electron inertia and write the equation of motion in the form

$$\frac{\partial N_e}{\partial z} = \frac{e}{T_e}\frac{\partial \phi}{\partial z} - \frac{e^2}{M\omega_i\omega}\left(\frac{\partial}{\partial z} + \frac{k}{\omega}\frac{\partial}{\partial t}\right)\frac{|E|^2}{T_e}. \tag{9}$$

The equations of longitudinal motion of the ions and dust are

$$\frac{\partial V_z}{\partial t} = -\frac{e}{M}\frac{\partial \phi}{\partial z} + \frac{e^2}{M^2\omega(\omega_i-\omega)}\left(\frac{\partial}{\partial z} + \frac{k}{\omega}\frac{\omega_i}{\omega_i-\omega}\frac{\partial}{\partial t}\right)|E|^2, \tag{10}$$

$$\frac{\partial w_z}{\partial t} = -\frac{Ze}{m_d}\frac{\partial \phi}{\partial z} + \frac{Z^2e^2}{m_d^2\omega(\omega_d+\omega)}\left(\frac{\partial}{\partial z} + \frac{k}{\omega}\frac{\omega_d}{\omega_d+\omega}\frac{\partial}{\partial t}\right)|E|^2. \tag{11}$$

Last terms in the r.h.s. of equations (9)-(11) describe the ponderomotive force acting on the particles by the moving wave.

With using expansion over a small parameter $\omega/\omega_i \ll 1$ one can transform equation of motion of the ions and dust (10)-(11) in the reference frame moving with the wave group velocity to the form

$$-u\frac{\partial V_z}{\partial \xi} = -\frac{e}{M}\frac{\partial \phi}{\partial \xi} + \frac{f+F}{M}, \tag{12}$$

$$- u\frac{\partial w_z}{\partial \xi} = -\frac{e}{m_e}\frac{\partial \phi}{\partial \xi} - \frac{Zf - m_d F/M}{m_d}, \tag{13}$$

where

$$f = \frac{\partial W_1}{\partial \xi} = \frac{e^2}{M\omega\omega_i}\left(1 - \frac{ku}{\omega}\right)\frac{\partial |E|^2}{\partial \xi},$$
$$F = \frac{\partial W_2}{\partial \xi} = \frac{e^2}{M\omega_i^2}\left(1 - \frac{2ku}{\omega}\right)\frac{\partial |E|^2}{\partial \xi}.$$

After integration of (9) and (12)-(13) one can find the density perturbations as

$$N_e = \frac{e\phi - W_1}{T_e},\ N_i = \frac{\mathrm{e}\,\phi - W_1 - W_2}{Mu^2},$$
$$N_d = \frac{-Ze\phi + ZW_1 - m_d W_2/M}{m_d u^2}.$$

Then the quasineutrality condition

$$n_{i0}N_i = n_{e0}N_e + Zn_{d0}N_d, \tag{14}$$

gives the following solution for the Coulomb potential

$$\frac{e\phi}{T_e} = \frac{W_1}{T_e} - \frac{W_2}{M(u^2 - c_s^2)}, \tag{15}$$

and the explicit solutions for the density perturbations

$$N_e = -\frac{W_2}{M(u^2 - c_s^2)},\ N_i = -\frac{W_2}{Mu^2}\left(1 + \frac{T_e/M}{u^2 - c_s^2}\right),\ N_d \simeq -\frac{W_2}{Mu^2}, \tag{16}$$

here $c_s^2 n_{i0}T_e/n_{e0}M$ is the square of the ion-sound dust speed. The longitudinal velocities are connected through the continuity equations with the density perturbations by the equations

$$v_{zk} = uN_k. \tag{17}$$

With accounting of equations (16) and (17) in the last terms in equation (7) one can reduce it to the following form for the new variable $b = B/B_0 =$

$kcE/\omega B_0$

$$\pm\, i\frac{\partial b}{\partial \tau} + \frac{u'}{2}\frac{\partial^2 b}{\partial \xi^2} - \frac{1}{2}kv_A\frac{u^2 - \rho_d c_s^2/\rho}{u^2 - c_s^2}|b|^2 b = 0. \tag{18}$$

The Lighthill criterion for the modulational instability fulfils when the signs of u' and of the nonlinear term in (18) coincide. Thus, for Alfvén wave with $u' > 0$ modulational unstability takes place whenever the conditions $c_s^2\rho_d/\rho < u^2 < c_s^2$ hold. In the interstellar plasma with $\rho_d/\rho \sim 0.01$ both these conditions are valid. For the right-polarized Alfvén waves with $u' < 0$ modulational instability can develop only in one of the two cases: either $u^2 > c_s^2$ or $u^2 < c_s^2\rho_d/\rho$, which hardly can hold in the conditions of the interstellar matter. Therefore, left-polarized Alfvén waves with the amplitude higher than a critical value experience in the interstellar plasma modulational instability which results in formation of a sequence of Alfvén solitons. In the limit $u \to c_s$ the free term in (18) becomes singular, which means that the quasineutrality condition is not valid anymore.

Using the standard procedure we represent the magnetic field as $b = b_\perp e^{i\Phi}$, where $b_\perp = b_\perp(\zeta - vt)$, $\Phi = \Phi(\zeta - v_1 t)$, what means that the envelope solution we are seeking for drifts slowly (i.e. $v \ll u$) in the reference frame moving with the group velocity, while the phase Φ drifts with the velocity $v_1 \ll u$. Separating the real and imaginary parts of (18) we arrive at

$$\begin{aligned} &\frac{\partial^2 b_\perp}{\partial \zeta^2} - \frac{v(v-2v_1)}{u'^2}b_\perp + \frac{2\alpha}{u'}b_\perp^3 = 0,\\ &\frac{\partial \Phi}{\partial \zeta} = \frac{v}{u'}, \end{aligned} \tag{19}$$

where

$$\alpha = -\frac{\omega}{4}\frac{u^2 - c_s^2\rho_d/\rho}{u^2 - c_s^2}. \tag{20}$$

The solution of (19) is

$$b_\perp = b_m \mathrm{ch}^{-1}\left(\frac{\zeta - vt}{\Lambda}\right), \tag{21}$$

where $\Lambda = \sqrt{u'/\alpha b_m^2}$, and v_1 can be found as $v_1 = v(1 - \alpha u' b_m^2/v^2)/2$. In general, the solution (21) is determined by the two free parametes: b_m and v. It is readily seen that collisions of dust particles with ions and atoms are unimportant on the scales of interest. Indeed, the characteristic length of the

soliton (21) for $\omega \ll \omega_d$ can be estimated as $\Delta\zeta \gg r_{Ad} b_m^{-1}$, or for typical parameters in the interstellar plasma $\Delta\zeta \gg 10^{15} b_m^{-1}$ cm, which on the lower end is much less than the drag free path of dust particles: $\ell \sim 3 \times 10^{19} n^{-1}$ cm, where the mass ratio $m_d/m \sim 10^{10}$ and mean grain radius $a = 0.1\mu$m are assumed.

4 Longitudinal and transversal acceleration of dust particles

In the laboratory reference frame longitudinal acceleration of dust particles is determined by the last equation in (16) and (17): $|V_{dz}| \simeq u|b|^2/2$, so that within the validity of the approximation the amplitude of dust velocity on nonlinear Alfvén wave can be of several percents of the Alfvén speed, which in the conditions of the interstellar plasma can vary from $v_A \sim 3$ km s^{-1} in molecular gas ($\rho \sim 10^{-22}$ g cm^{-3}, $B \sim 10\mu$G) to ~ 10 km s^{-1} in diffuse HI phasei ($\rho \sim 10^{-24}$ g cm^{-3}, $B \sim 3\mu$G). Therefore, reasonable conservative estimate of dust acceleration along magnetic lines in such environments may be of 0.3 to 1 km s^{-1}, respectively, which however depends on exact value of the soliton amplitude. This is much lower than the estimate obtained in [6]. Similarly, the dust velocity in perpendicular direction due to acceleration by the soliton can be estimated as $|V_{d\perp}| \simeq \omega |b_\perp|/k \sim u|b|$, which is therefore a factor of b_m^{-1} larger than acceleration in the longitudinal direction.

The acceleration of dust grains by a regular low-frequency Alfvén soliton ($\omega \ll \omega_d$) is connected with the ponderomotive force as in [6], and differs qualitatively from the mechanism described in [3], where translational heating of dust particles is associated with stochastic acceleration of dust particles by MHD waves with $\omega \sim \omega_d$ through cyclotron resonant interactions. Indeed, it is readily seen that dust particle moves under the action of soliton wave only within the soliton, while it turns to the rest outside the wave, e.i. at $\zeta \to -\infty$, and the only result of the action of the soliton is a shift of a particle by $\Delta\zeta \sim \Lambda$.

The analysis of possible turbulent acceleration of dust particles by a random set of low-frequence Alfvén solitons is out of the scape of our paper. One can speculate only that as far as translational heating of dust particles by low-frequency MHD waves is concerned, one can connect it with an ensemble of solitons passing randomly through a given volume of the interstellar space. In this picture the effective translational temperature is determined by the spectrum of low-frequency MHD solitons.

5 Summary

In this paper we considered acceleration of charged dust particles by nonlinear low-frequency MHD waves under the conditions typical for interstellar plasma. We have shown that contrary to the case of intermediate frequencies $\omega_d \ll \omega \ll \omega_i$ described in [6], the acceleration efficiency is strongly limited by the amplitude of the nonlinear wave: while transversal velocities of dust particles $V_{d\perp}$ are proportional to the amplitude of the soliton, the longitudinal velocities V_{dz} are of the second order of the amplitude. As a consequence, dust particles can be accelerated by such low-frequency waves to the velocities of few to tens percents of the Alfvén speed.

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Assuming $m_d\rho_d/Z_d m_i \rho_i \gg 1$ and $\omega \ll \rho_d \omega_d/\rho$, $\rho = \rho_i + \rho_d$, one can get from here

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$$\frac{e\phi}{T_e} = \frac{W_1}{T_e} - \frac{W_2}{M(u^2 - c_s^2)}, \tag{15}$$

and the explicit solutions for the density perturbations

$$N_e = -\frac{W_2}{M(u^2 - c_s^2)},\ N_i = -\frac{W_2}{Mu^2}\left(1 + \frac{T_e/M}{u^2 - c_s^2}\right),\ N_d \simeq -\frac{W_2}{Mu^2}, \tag{16}$$

here $c_s^2 n_{i0}T_e/n_{e0}M$ is the square of the ion-sound dust speed. The longitudinal velocities are connected through the continuity equations with the density perturbations by the equations

$$v_{zk} = uN_k. \tag{17}$$

With accounting of equations (16) and (17) in the last terms in equation (7) one can reduce it to the following form for the new variable $b = B/B_0 =$

$kcE/\omega B_0$

$$\pm\, i\frac{\partial b}{\partial \tau} + \frac{u'}{2}\frac{\partial^2 b}{\partial \xi^2} - \frac{1}{2}kv_A\frac{u^2-\rho_d c_s^2/\rho}{u^2-c_s^2}|b|^2 b = 0. \tag{18}$$

The Lighthill criterion for the modulational instability fulfils when the signs of u' and of the nonlinear term in (18) coincide. Thus, for Alfvén wave with $u' > 0$ modulational unstability takes place whenever the conditions $c_s^2\rho_d/\rho < u^2 < c_s^2$ hold. In the interstellar plasma with $\rho_d/\rho \sim 0.01$ both these conditions are valid. For the right-polarized Alfvén waves with $u' < 0$ modulational instability can develop only in one of the two cases: either $u^2 > c_s^2$ or $u^2 < c_s^2\rho_d/\rho$, which hardly can hold in the conditions of the interstellar matter. Therefore, left-polarized Alfvén waves with the amplitude higher than a critical value experience in the interstellar plasma modulational instability which results in formation of a sequence of Alfvén solitons. In the limit $u \to c_s$ the free term in (18) becomes singular, which means that the quasineutrality condition is not valid anymore.

Using the standard procedure we represent the magnetic field as $b = b_\perp e^{i\Phi}$, where $b_\perp = b_\perp(\zeta - vt)$, $\Phi = \Phi(\zeta - v_1 t)$, what means that the envelope solution we are seeking for drifts slowly (i.e. $v \ll u$) in the reference frame moving with the group velocity, while the phase Φ drifts with the velocity $v_1 \ll u$. Separating the real and imaginary parts of (18) we arrive at

$$\begin{aligned} &\frac{\partial^2 b_\perp}{\partial \zeta^2} - \frac{v(v-2v_1)}{u'^2}b_\perp + \frac{2\alpha}{u'}b_\perp^3 = 0, \\ &\frac{\partial \Phi}{\partial \zeta} = \frac{v}{u'}, \end{aligned} \tag{19}$$

where

$$\alpha = -\frac{\omega}{4}\frac{u^2 - c_s^2\rho_d/\rho}{u^2-c_s^2}. \tag{20}$$

The solution of (19) is

$$b_\perp = b_m \mathrm{ch}^{-1}\left(\frac{\zeta - vt}{\Lambda}\right), \tag{21}$$

where $\Lambda = \sqrt{u'/\alpha b_m^2}$, and v_1 can be found as $v_1 = v(1 - \alpha u' b_m^2/v^2)/2$. In general, the solution (21) is determined by the two free parametes: b_m and v. It is readily seen that collisions of dust particles with ions and atoms are unimportant on the scales of interest. Indeed, the characteristic length of the

soliton (21) for $\omega \ll \omega_d$ can be estimated as $\Delta\zeta \gg r_{Ad} b_m^{-1}$, or for typical parameters in the interstellar plasma $\Delta\zeta \gg 10^{15} b_m^{-1}$ cm, which on the lower end is much less than the drag free path of dust particles: $\ell \sim 3 \times 10^{19} n^{-1}$ cm, where the mass ratio $m_d/m \sim 10^{10}$ and mean grain radius $a = 0.1\mu$m are assumed.

4 Longitudinal and transversal acceleration of dust particles

In the laboratory reference frame longitudinal acceleration of dust particles is determined by the last equation in (16) and (17): $|V_{dz}| \simeq u|b|^2/2$, so that within the validity of the approximation the amplitude of dust velocity on nonlinear Alfvén wave can be of several percents of the Alfvén speed, which in the conditions of the interstellar plasma can vary from $v_A \sim 3$ km s^{-1} in molecular gas ($\rho \sim 10^{-22}$ g cm^{-3}, $B \sim 10\mu$G) to ~ 10 km s^{-1} in diffuse HI phasei ($\rho \sim 10^{-24}$ g cm^{-3}, $B \sim 3\mu$G). Therefore, reasonable conservative estimate of dust acceleration along magnetic lines in such environments may be of 0.3 to 1 km s^{-1}, respectively, which however depends on exact value of the soliton amplitude. This is much lower than the estimate obtained in [6]. Similarly, the dust velocity in perpendicular direction due to acceleration by the soliton can be estimated as $|V_{d\perp}| \simeq \omega|b_\perp|/k \sim u|b|$, which is therefore a factor of b_m^{-1} larger than acceleration in the longitudinal direction.

The acceleration of dust grains by a regular low-frequency Alfvén soliton ($\omega \ll \omega_d$) is connected with the ponderomotive force as in [6], and differs qualitatively from the mechanism described in [3], where translational heating of dust particles is associated with stochastic acceleration of dust particles by MHD waves with $\omega \sim \omega_d$ through cyclotron resonant interactions. Indeed, it is readily seen that dust particle moves under the action of soliton wave only within the soliton, while it turns to the rest outside the wave, e.i. at $\zeta \to -\infty$, and the only result of the action of the soliton is a shift of a particle by $\Delta\zeta \sim \Lambda$.

The analysis of possible turbulent acceleration of dust particles by a random set of low-frequence Alfvén solitons is out of the scape of our paper. One can speculate only that as far as translational heating of dust particles by low-frequency MHD waves is concerned, one can connect it with an ensemble of solitons passing randomly through a given volume of the interstellar space. In this picture the effective translational temperature is determined by the spectrum of low-frequency MHD solitons.

5 Summary

In this paper we considered acceleration of charged dust particles by nonlinear low-frequency MHD waves under the conditions typical for interstellar plasma. We have shown that contrary to the case of intermediate frequencies $\omega_d \ll \omega \ll \omega_i$ described in [6], the acceleration efficiency is strongly limited by the amplitude of the nonlinear wave: while transversal velocities of dust particles $V_{d\perp}$ are proportional to the amplitude of the soliton, the longitudinal velocities V_{dz} are of the second order of the amplitude. As a consequence, dust particles can be accelerated by such low-frequency waves to the velocities of few to tens percents of the Alfvén speed.

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